The multi-depot VRP with vehicle interchanges

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Abstract.
This work introduces a new variant of the vehicle routing problem with multiple depots. In this problem, both, capacity constraints on the vehicles and length constraints on the routes of the drivers are imposed. To favor a better utilization of the available capacities and working times of the drivers, it is allowed to combine pairs of routes at predefined interchange points, where two drivers can exchange their vehicles so that, even if each driver must return to his home depot at the end of the journey, vehicles can follow paths from one depot to another one.

Keywords: Vehicle routing, Multiple depots, Interchange locations.

1 Problem definition

Vehicle routing is a core class of problems in combinatorial optimization. Among the most classical variants we find the capacitated vehicle routing problem (CVRP) where the goal is to satisfy the demands of a set of customers by performing closed routes from a given depot, each satisfying two conditions: the total demand delivered by a vehicle cannot exceed the vehicle capacity, and the total duration of any route must be below a prespecified limit.

A natural extension of this problem is to consider situations where several depots are available [1, 2]; this situation is specially common when demand points are scattered in large geographical areas. In this case, additionally to the above conditions, one must ensure that each route finishes at the depot where it started. This extra condition is typically associated with the driver working conditions; specially when large distribution areas are considered, asking drivers to perform open routes may result in extra labor costs which cannot be compensated by possible savings in the distribution costs.

In this talk we present a new variant of this problem where this last condition can be partially relaxed. In the multi-depot vehicle routing problem with vehicle interchanges (MDVRPVI), additionally to the customers and the depots, an extra set of locations is also considered. These locations, known as interchange points can be used by pairs of drivers to meet during their routes and interchange their vehicles. In this case, although drivers perform closed routes that start and end at the same depot, vehicles can follow paths that start at one depot, and finish at a different one. Indeed, this type of policy is already used in other transportation modes where drivers or crew routes are disaggregated from vehicle routes to allow the drivers end their journey at their home depot, without excessively conditioning the shape of the vehicle routes. In the case of the CVRP, additionally, depending on the distribution of the customers and their demands, if the interchange points are well placed, this policy can yield significant cost savings, since it allows for more flexible combinations of route durations and vehicle capacity utilization.

2 MDVRPDI solutions

The main extra difficulty in formulating the MDVRPVI is the need for synchronization of the pairs of routes that interchange their vehicles. This requires considering explicitly the direction in which routes are performed, regardless the distances are symmetric or not.

Taking this into account, we propose a first MIP formulation of the MDVRPVI, based on the classical three-index vehicle flow formulation of the CVRP [3]. Although, as it is well known, this type of formulations present awkward symmetry problems, this first formulation allows us to obtain optimal solutions
to small instances of the MDVRPVI. Based on these optimal solutions we can study empirically the solutions of the MDVRPVI. In particular, we study $i$) the impact of the locations of the interchange points and the demand pattern on the solution structure, and $ii$) the savings that can be attained by introducing this policy to the multi-depot CVRP.

Moreover, we also study the savings that the interchange of vehicles can yield from a theoretical point of view by proving that these savings cannot exceed 50% of the solution cost. Furthermore, we prove that this bound is tight by introducing a family of instances where this bound is attained.

References

